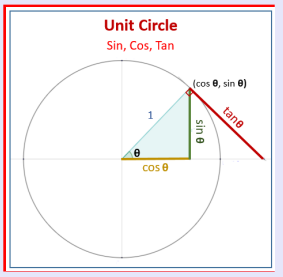
Trigonometry

Lecture 39



Feb 19-8:47 AM

Solve $3\tan x + 1 = 0$, give General Solution in degrees.

$\tan x = -\frac{1}{3}$

QII, QIV

R.A. $\tan^{-1}\left(\frac{1}{3}\right) \approx 18^\circ$

QII $\quad x = 180^\circ - 18^\circ + K\cdot 180^\circ$

QIV $\quad x = 360^\circ - 18^\circ + K\cdot 180^\circ$

$x = 162^\circ + K\cdot 180^\circ$

$x = 342^\circ + K\cdot 180^\circ$

Nov 7-10:26 AM

Solve $\sin^2 2\theta = 2\sin 2\theta + 3$, give general solution in radians.

$\sin^2 2\theta - 2\sin 2\theta - 3 = 0$

$(\sin 2\theta + 1)(\sin 2\theta - 3) = 0$

$\sin 2\theta + 1 = 0$ $\qquad$ $\sin 2\theta - 3 = 0$

$\sin 2\theta = -1$ $\qquad$ $\sin 2\theta = 3$

$\uparrow$ $\qquad$ Has no Solution

$\frac{3\pi}{2}$

$2\theta = \frac{3\pi}{2} + K \cdot 2\pi$

$\boxed{\theta = \frac{3\pi}{4} + K \cdot \pi}$

Nov 7-10:31 AM

Find all solutions of $\sqrt{3}\tan\frac{x}{2} - 1 = 0$.

$\tan\frac{x}{2} = \frac{1}{\sqrt{3}}$ $\qquad$ $\tan\frac{x}{2} = \frac{\sqrt{3}}{3}$

QI, QIII

QI $\quad \frac{x}{2} = \frac{\pi}{6} + K \cdot \pi$ $\qquad$ RA. $\frac{\pi}{6}$

multiply by 2

$2 \cdot \frac{x}{2} = 2 \cdot \frac{\pi}{6} + K \cdot 2\pi$ $\qquad$ $\boxed{x = \frac{\pi}{3} + 2K\pi}$

QIII $\quad \frac{x}{2} = \pi + \frac{\pi}{6} + K \cdot \pi$ $\qquad$ $\frac{x}{2} = \frac{7\pi}{6} + K \cdot \pi$

$\boxed{x = \frac{7\pi}{3} + 2K\pi}$

Give all Solutions on $[0, 4\pi)$

$K=0 \rightarrow \frac{\pi}{3}, \frac{7\pi}{3}$ $\qquad$ $K=1$ $\quad \frac{\pi}{3} + 2\pi = \cancel{\frac{7\pi}{3}}$

$\frac{7\pi}{3} + 2\pi = \frac{13\pi}{3}$

$= \frac{12\pi}{3} + \frac{\pi}{3}$

$4\pi + \frac{\pi}{3}$

outside of $[0, 4\pi)$

$\left\{\frac{\pi}{3}, \frac{7\pi}{3}\right\}$

Nov 7-10:36 AM

$\cos\theta - \sin\theta = \sqrt{2}\sin\frac{\theta}{2}$

$-2 \neq 2$

$(-2)^2 = 2^2$

$4 = 4$ ✓

$(\cos\theta - \sin\theta)^2 = \left(\sqrt{2}\sin\frac{\theta}{2}\right)^2$

$(\cos\theta - \sin\theta)(\cos\theta - \sin\theta) = 2\left[\sin\frac{\theta}{2}\right]^2$

$\cos^2\theta - \cos\theta\sin\theta - \sin\theta\cos\theta + \sin^2\theta = 2\left[\pm\sqrt{\frac{1-\cos\theta}{2}}\right]^2$

$1 - 2\sin\theta\cos\theta = 2 \cdot \frac{1-\cos\theta}{2}$

$1 - 2\sin\theta\cos\theta = 1 - \cos\theta$

$-2\sin\theta\cos\theta + \cos\theta = 0$

$2\sin\theta\cos\theta - \cos\theta = 0$

$\cos\theta[2\sin\theta - 1] = 0$

$\cos\theta = 0$

$\pi/2, 3\pi/2$

$90°, 270°$

$\sin\theta = \frac{1}{2}$ R.A. $\frac{\pi}{6}$

$30°, 150°$

$\frac{\pi}{6}, \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Nov 7-10:45 AM

~~90°~~, 270° ✓

✓ 30°, 150°

Possible Solutions

$\cos\theta - \sin\theta = \sqrt{2}\sin\frac{\theta}{2}$

$\theta = 90°$

$\cos 90° - \sin 90° = \sqrt{2}\sin 45°$

$0 \quad 1 \quad \frac{\sqrt{2}}{2}$

$\theta = 270°$

$\cos 270° - \sin 270° = \sqrt{2}\sin 135°$

$0 \quad -1$

$-1 = \sqrt{2} \cdot \frac{\sqrt{2}}{2}$

$-1 = \frac{\sqrt{4}}{2} = \frac{2}{2} = 1$

$1 = \sqrt{2} \cdot \frac{\sqrt{2}}{2}$

$1 = 1$ ✓

$\theta = 30°$

$\cos 30° - \sin 30° = \sqrt{2}\sin 15°$

$\frac{\sqrt{3}}{2} - \frac{1}{2} = \sqrt{2} \cdot \frac{\sqrt{6}-\sqrt{2}}{4}$

$\frac{\sqrt{3}-1}{2} = \frac{\sqrt{12}-\sqrt{4}}{4} = \frac{\sqrt{4}\sqrt{3}-\sqrt{4}}{4}$

$= \frac{2\sqrt{3}-2}{4}$

$= \frac{2(\sqrt{3}-1)}{4}$

$= \frac{\sqrt{3}-1}{2}$

$\sin 15° = \sin(45° - 30°)$

$= \sin 45°\cos 30° - \cos 45°\sin 30°$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

Nov 7-10:55 AM

$\theta = 150^\circ$ Does not work

$\cos 150^\circ - \sin 150^\circ = \sqrt{2}\sin 75^\circ$

RA 30°

$-\frac{\sqrt{3}}{2} - \frac{1}{2} = \sqrt{2}\sin(45^\circ + 30^\circ)$

$= \sqrt{2}\left[\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ\right]$

$-\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \sqrt{2}\left[\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2}\right]$

$= \frac{\sqrt{12} + \sqrt{4}}{4}$

Final Ans $= \frac{2\sqrt{3} + 2}{4}$

$\{30^\circ, 270^\circ\}$ $= \frac{2(\sqrt{3}+1)}{4} = \frac{\sqrt{3}+1}{2}$

Nov 7-11:04 AM

Solve $\frac{1-\cos x}{1+\cos x} = 3 = \frac{3}{1}$

Cross-Multiply

$3(1+\cos x) = 1 - \cos x$

$3 + 3\cos x = 1 - \cos x$

$3\cos x + \cos x = 1 - 3$

$4\cos x = -2$

$\cos x = -\frac{1}{2}$

QII, QIII

R.A. 60°

QII $x = 180^\circ - 60^\circ = \boxed{120^\circ}$

QIII $x = 180^\circ + 60 = \boxed{240^\circ}$

Nov 7-11:09 AM

Solve $\tan 2x + \sec 2x = \sqrt{3}$

$(A+B)^2 = A^2+2AB+B^2$

Square both sides

$(\tan 2x + \sec 2x)^2 = (\sqrt{3})^2$

$\tan^2 2x + 2\tan 2x \sec 2x + \sec^2 2x = 3$

$1 + \tan^2\alpha = \sec^2\alpha$

$\tan^2 2x + 2\tan 2x \sec 2x + (1) + \tan^2 2x = 3$

$2\tan^2 2x + 2\tan 2x \sec 2x = 2$

$\tan^2 2x + \tan 2x \sec 2x = 1$

$\frac{\sin^2 2x}{\cos^2 2x} + \frac{\sin 2x}{\cos 2x} \cdot \frac{1}{\cos 2x} = 1$

LCD $= \cos^2 2x$

$\sin^2 2x + \sin 2x = \cos^2 2x$

$\sin^2 2x + \sin 2x = 1 - \sin^2 2x$

$2\sin^2 2x + \sin 2x - 1 = 0$

$(\sin 2x + 1)(2\sin 2x - 1) = 0$

$\sin 2x = -1$ $\quad$ $\sin 2x = \frac{1}{2}$

$2x = 270^\circ + K\cdot 360^\circ$ $\quad$ 270° $\quad$ $30^\circ, 150^\circ$

$x = 135^\circ + K\cdot 180^\circ$

$2x = 30^\circ + K\cdot 360^\circ$

$x = 15^\circ + K\cdot 180^\circ$ $\quad$ $x = 75^\circ + K\cdot 180^\circ$

$2x = 150^\circ + K\cdot 360^\circ$

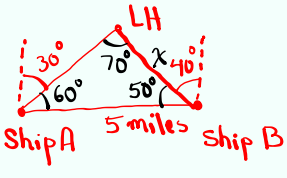
Nov 7-11:15 AM

Class QZ 8

Ship A has bearing of N 30° E for a light house

Ship B is 5 miles east of ship A and has a bearing of N 40° W for the same lighthouse.

How far is Ship B from the lighthouse. Drawing Required.



$\frac{\sin 70^\circ}{5} = \frac{\sin 60^\circ}{x}$

$x = \frac{5\sin 60^\circ}{\sin 70^\circ}$

$x \approx 4.6$ miles

$x \approx 5$ miles

Nov 7-11:30 AM